Directions: This exam consists of 8 questions. In order to pass the exam, you must answer each question. Where there have been multiple professors teaching the same class, we have provided a question from each (A. and B.). You should select the question most familiar for you, but don’t answer both. If you do answer both, only A. will be graded.

Do not write your name on your answer sheet or on any exam page other than this cover sheet. You have been assigned an identification letter below. Write this identification letter on each of your answer pages and on each of the test sheets.

Please note that we will not accept any answer sheet pages with your name on them; if your name is on the sheet we will not grade it.

Your identification letter for this exam is: _______________________

Academic Honesty Statement: All students must comply with University policies on academic integrity. Any student violating these policies, as defined on pages 262-263 of the Graduate Catalog, will be referred to the University administration for disciplinary action. Sanctions for academic dishonesty include, but are not limited to, failure of this exam, suspension, or expulsion from the University. By signing below you certify that the answers you place on this exam are your own and that you have not received help from others.

I hereby certify that the work on this exam is my own. I also certify that I am aware that the test cannot be turned in after 4:00 pm and that I must place my exam identification letter on my exam sheets.

Student name (print) _________________________________

Student Signature: _________________________________
1. There were two professors who taught FINN/ECON6203. We have provided one question from each professor below. Please answer either question A OR question B below. Do not answer both A AND B.

A. **Show the details of your work! Answer all parts.**
Consider a two-period Binomial model in which time $t = 0, 1, 2$, and each time period represents one year. There are two basic securities in the market. The first one is a riskless asset with constant interest rate 2% (annually compounding). The second one is a risky asset, and at each time period, this asset's price can either move up 30% or move down 30%. The time zero price of the risky asset is $10. One investment bank XYZ sells a European-type derivative with two years to maturity, and this derivative structured product can be exercised only at time $t=2$. When the derivative is exercised at time $t$, the owner of the derivative receives, from XYZ,

where $S_0$ and $S_t$ are the risky asset's price at time 0 and time $t$, respectively,

a) Determine the fair price of this derivative at time $t=0$. In other words, ignoring transaction costs, how much XYZ should charge to the buyer at time $t=0$ to break even?

b) Assume XYZ charges $4 for this derivative. Does XYZ make money for sure without implementing a hedging strategy? If necessary, construct a hedging strategy, or equivalently, a hedging portfolio by these two basic assets, from time $t=0$ and $t=1$.

B. **Show the details of your work! Answer all parts.**
Consider a one-period market model with two times, $t = 0$ and $t = T > 0$, $K = 3$ states and $N = 3$ securities with time $T$ payoffs given by the columns of

$$D = \begin{bmatrix} 23 & 12 & 10 \\ 30 & 21 & 10 \\ 46 & 27 & 10 \end{bmatrix}$$

and price vector

$$p = \begin{bmatrix} 27 \\ 17 \\ 9 \end{bmatrix}.$$

a) Is this market complete? Explain why or why not.
b) Does the price system admit arbitrage? Explain why or why not.
c) Does there exist a state price vector? If so find one. Is it unique?
d) Does there exist an equilibrium price measure? If so find one. Is it unique?
e) What is the equilibrium price of a security whose payoff at time 1 is the difference of the payoffs of Security 1 and Security 2 (that is, payoff = columns 1 of $D$ – column 2 of $D$)?
2. There were two professors who taught FINN/ECON6219. We have provided one question from each professor below. Please answer either question A OR question B below. Do not answer both A AND B.

A. Show the details of your work!

Given a stock index monthly return time series data and you want to check whether the market is efficient or not. You decide to perform a unit-root test. Present your null hypothesis, alternative hypothesis and the decision rule.

B. Show the details of your work! Answer all parts.

The three ARCH/GARCH models below were estimated for one year of daily S&P 500 Index returns. In each model $\varepsilon_t$ is a Gaussian white noise series.

a) Identify the type and order of each of the ARCH/GARCH model below.

b) Based on the given information, determine which model is the best for the given time series and explain why.

Model I

$$r_i = -0.000262 + a_i$$

$$a_i = \sigma_i \varepsilon_t$$

$$\sigma_i^2 = 1.301714 - 0.000005a_{i-1}^2$$

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>-0.000262</td>
<td>0.00075432</td>
<td>-0.3473</td>
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<tr>
<td>Cst(V)</td>
<td>1.301714</td>
<td>0.13065</td>
<td>9.963</td>
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<tr>
<td>ARCH(Alpha1)</td>
<td>-0.000005</td>
<td>0.060717</td>
<td>-0.00</td>
</tr>
</tbody>
</table>

AIC $-6.084931$

Q-Statistics on Standardized Residuals [P-Values in brackets]

| Q(10) | 7.87960 [0.6405963] |
| Q(15) | 13.6750 [0.5503078]  |
| Q(20) | 14.6565 [0.7957146]  |

Q-Statistics on Squared Standardized Residuals [P-Values in brackets]

| Q(10) | 55.9756 [0.0000000] |
| Q(15) | 65.7094 [0.0000000] |
| Q(20) | 81.4076 [0.0000000] |
**Model II**

\[ r_t = -0.000952 + a_t \]

\[ a_t = \sigma_t \varepsilon_t \]

\[ \sigma_t^2 = 0.036885 + 0.104814 a_{t-1}^2 + 0.870776 \sigma_{t-1}^2 \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>-0.000952</td>
<td>0.00060227</td>
<td>-1.581</td>
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<tr>
<td>Cst(V)</td>
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<td>0.020453</td>
<td>1.803</td>
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<tr>
<td>ARCH(Alpha1)</td>
<td>0.104814</td>
<td>0.031052</td>
<td>3.375</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.870776</td>
<td>0.030584</td>
<td>28.47</td>
</tr>
</tbody>
</table>

AIC = -6.216553

**Q-Statistics on Standardized Residuals [P-Values in brackets]**

- Q(10) = 7.24759 [0.7018829]
- Q(15) = 10.9317 [0.7574193]
- Q(20) = 11.5096 [0.9319230]

**Q-Statistics on Squared Standardized Residuals [P-Values in brackets]**

- Q(10) = 19.7115 [0.0114845]
- Q(15) = 24.7515 [0.0248805]
- Q(20) = 29.5278 [0.0422976]

**Model III**

\[ r_t = -0.001171 + 2.263867 \sigma_t^2 + a_t \]

\[ a_t = \sigma_t \varepsilon_t \]

\[ \sigma_t^2 = 0.036875 + 0.104756 a_{t-1}^2 + 0.870846 \sigma_{t-1}^2 \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>-0.001171</td>
<td>0.0011806</td>
<td>-0.9920</td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.036875</td>
<td>0.020321</td>
<td>1.815</td>
</tr>
<tr>
<td>ARCH(Alpha1)</td>
<td>0.104756</td>
<td>0.030945</td>
<td>3.385</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.870846</td>
<td>0.030330</td>
<td>28.71</td>
</tr>
<tr>
<td>ARCH-in-mean(var)</td>
<td>2.263867</td>
<td>10.549</td>
<td>0.2146</td>
</tr>
</tbody>
</table>

AIC = -6.208769

**Q-Statistics on Standardized Residuals [P-Values in brackets]**

- Q(10) = 7.40316 [0.6869142]
<table>
<thead>
<tr>
<th>Q(15)</th>
<th>Q(20)</th>
<th>P-Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.0453</td>
<td>11.6373</td>
<td>[0.7493795, 0.9279958]</td>
</tr>
</tbody>
</table>

Q-Statistics on Squared Standardized Residuals [P-Values in brackets]

<table>
<thead>
<tr>
<th>Q(10)</th>
<th>Q(15)</th>
<th>Q(20)</th>
<th>P-Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.0398</td>
<td>24.8263</td>
<td>29.7673</td>
<td>[0.0101866, 0.0243264, 0.0397707]</td>
</tr>
</tbody>
</table>

3. Answer the following question related to MATH6203 material. Show the details of your work!

**Problem on Stochastic Calculus for Finance.** The Vasicek Short Rate model is

\[ dR_t = (\alpha - \beta R_t)dt + \sigma dW_t, \]

where \( \alpha, \beta, \sigma \) are constants. Use Itô-Doebelin formula to find the differential \( d(e^{\beta t} R_t) \), and then find the solution to \( R_t \). What is the distribution of \( R_t \) for fixed time \( t \)?

4. There were two professors who taught MATH6202. We have provided one question from each professor below. Please answer either question A OR question B below. Do not answer both A AND B.

**A. Show the details of your work!**

**Problem on PDE for Finance.** Suppose the stock prices process follows the dynamics

\[ dS_t = S_t(rdt + \sigma dW_t + \gamma d\tilde{N}_t), \]

where \( r, \sigma, \gamma \) are constants, \( \tilde{W}_t \) is a Brownian motion, and \( \tilde{N}_t \) is a compensated Poisson process with intensity \( \lambda \) under a risk neutral probability measure \( \tilde{P} \). Derive the pricing integral differential equation with proper boundary condition for a European option with payoff

\[ V_T = f(S_T). \]

Is this a complete market model or incomplete market model?

**4. B. Partial Differential Equations for Finance on next page**
4. B. Show the details of your work!

1. Suppose that we already have the following result:

   Let \( L_{S,t} \) be an operator in an option problem in the form:

   \[
   L_{S,t} = a(S, t) \frac{\partial^2}{\partial S^2} + b(S, t) \frac{\partial}{\partial S} + c(S, t)
   \]

   and \( G(S, t) \) be the constraint function for an American option. Furthermore we assume that \( \frac{\partial G}{\partial t} + L_{S,t}G \) exists. Suppose \( V(S_t, t^*) = G(S, t^*) \) on an open interval \((A, B)\) on the \(S\)-axis. Let \( t = t^* - \Delta t \), where \( \Delta t \) is a sufficiently small positive number. Show the following conclusions: If for any \( S \in (A, B) \),

   \[
   \frac{\partial G}{\partial t}(S, t^*) + L_{S,t} G(S, t^*) + d(S, t^*) \geq 0,
   \]

   then the value \( V(S, t) \) determined by the equation

   \[
   \frac{\partial V}{\partial t}(S, t) + L_{S,t} V(S, t) + d(S, t) = 0
   \]

   satisfies the condition \( V(S, t) - G(S, t) \geq 0 \) on \((A, B)\); and if for any \( S \in (A, B) \),

   \[
   \frac{\partial G}{\partial t}(S, t^*) + L_{S,t} G(S, t^*) + d(S, t^*) < 0,
   \]

   then the equation

   \[
   \frac{\partial V}{\partial t}(S, t) + L_{S,t} V(S, t) + d(S, t) = 0
   \]

   cannot give a solution satisfying the condition \( V(S, t) - G(S, t) \geq 0 \) for any \( S \in (A, B) \).

Show the following result:

Suppose that for an American option, the constraint is \( G(S, t) \), its value at time \( t \) is \( V(S, t) \), and \( V(S, t) = G(S, t) \) on \((A, B)\). Assume that when \( V(S, t) \) is given as the value of a European option at \( t \), the value of the European option at \( t - \Delta t \) for a positive and very small \( \Delta t \) is \( v(S, t - \Delta t) \). In an open interval near \( S^* \in (A, B) \), \( v(S, t - \Delta t) < G(S, t - \Delta t) \), then for the American option a fair value at the point \( (S^*, t - \Delta t) \) should be \( G(S^*, t - \Delta t) \).
5. There were two professors who taught MATH6204. We have provided one question from each professor below. Please answer either question A OR question B below. Do not answer both A AND B.

A. Show the details of your work!

Problem: Explicit finite differences and trinomial trees. The Black-Scholes equation
\[ \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + r S \frac{\partial V}{\partial S} - r V = 0 \]
can be transformed into

\[ \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial x^2} + \left( r - \frac{1}{2} \sigma^2 \right) \frac{\partial V}{\partial x} - r V = 0, \]

where \( x = \log S, t \in [0, T], \) time \( t = 0 \) is the present time, and time \( T \) is the maturity date. \( V[S_t, t] \) is the time-\( t \) value of an European call and \( V[S_T, T] = \max\{S_T - K, 0\} \).

1. To compute \( V[S_0, 0] \), show how you can apply the explicit finite difference method (FDM) directly to the log-transformed partial differential equation given above.

2. From your answer to question (1), show that the explicit FDM is actually a direct application of trinomial trees.

B. Show the details of your work!

1. Show that if

\[
\max_{0 \leq m \leq M} \frac{x_m^2 (1 - x_m)^2 \sigma_m^2 \Delta \tau}{2 \Delta x^2} \leq \frac{1}{2},
\]

then for the scheme with variable coefficients

\[
\frac{u_{m+1}^n - u_m^n}{\Delta \tau} = \frac{1}{2} \left[ x_m (1 - x_m) \sigma_m^2 \Delta \tau \right] \frac{u_m^n}{\Delta x^2} - \frac{2u_m^n + u_{m-1}^n}{\Delta x} + (r - D_0)x_m (1 - x_m) \frac{u_{m+1}^n - u_{m-1}^n}{2\Delta x} - \left[ r (1 - x_m) + D_0 x_m \right] u_m^n,
\]

the condition \( |\lambda_y(x_m, \tau^n)| \leq 1 + O(\Delta \tau) \) is satisfied for any \( x_m = m/M \in [0, 1] \). (When you prove this result, you should derive the stability condition for explicit schemes by yourself.)
6. Answer all of the following questions related to FINN6211 material. Show the details of your work!

For each of the following statements, provide in your answer: (1) if you AGREE or DISAGREE with the statement, and (2) one to two sentences explanation for why (if you agree) or why not (if you disagree).

a) The modified duration of a Treasury bill equals its time to maturity.

b) The coupon yield curve always lies on or above the spot rate curve because the coupon payments enhance the yield to maturity of a coupon bond.

c) For a given coupon rate and initial yield, the longer the term to maturity, the greater the price volatility.

d) The total return is a measure of yield that incorporates an explicit assumption about the reinvestment risk.

e) Because of liquidity issue the yield of on-the-run Treasury bonds is usually higher than the yield of off-the-run Treasury bonds.

f) A barbell portfolio is always more risky than a bullet portfolio if they have the same market value and duration.

g) Suppose you have a short position in a 30-year 6% coupon bond and a long position in a zero-coupon bond with exactly the same market value and duration. If all spot rates fall by 20 basis points, your net position will rise in value.

h) Your assistant gave you the following table of spot and forward rates (all rates are annualized). You believe these numbers are all correct.

<table>
<thead>
<tr>
<th>t (years)</th>
<th>Spot Rate (%)</th>
<th>6-month Forward Rate ending at t (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4.00%</td>
<td>4.00%</td>
</tr>
<tr>
<td>1.0</td>
<td>4.25%</td>
<td>4.50%</td>
</tr>
<tr>
<td>1.5</td>
<td>4.42%</td>
<td>4.75%</td>
</tr>
<tr>
<td>2.0</td>
<td>4.35%</td>
<td>4.60%</td>
</tr>
</tbody>
</table>
A. Statistical Techniques in Finance. Show details of your work! Answer all parts.

1. a. If $X$ has uniform distribution on $[-a, a]$ for any $a > 0$ and $Y = X^2$, show that $X$ and $Y$ are uncorrelated.

b. $X_1, X_2, \ldots, X_n$ are independent identically distributed with density function $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$, $\lambda > 0$. Derive the maximum likelihood estimator of $\lambda$.

c. For the AR(1) model $X_t = \phi X_{t-1} + \epsilon_t$ where $\epsilon_t$ is WhiteNoise(0,1) and $|\phi| < 1$ derive the least squares estimator of $\phi$.

d. Let $\epsilon_t$ is WhiteNoise(0,1) and

$$a_t = \epsilon_t \sqrt{1 + 0.3a_{t-1}^2}$$

$$u_t = 0.45 + 0.5u_{t-1} + a_t$$

find the ACF of $a_t$. 
7. Continued. Students could select from Statistical Techniques or Econometrics to satisfy required coursework. If you answered A on the previous page, do not complete this section too. If you omitted A on the previous page, you must all parts of B below.

B. Econometrics. Show details of your work! Answer all parts.

There are competing stories concerning the real estate bubble and whether the bubble was predictable. The following is a time plot of the monthly Case-Shiller House Price Index for the city of Las Vegas, NV from January 1987 through April 2009:

![Time plot of Case-Shiller House Price Index](image)

The following three graphs depict the Dickey-Fuller test statistic calculated using a rolling three year (36 month) window.

7. B. Econometrics continued on next page
Exam Identification Letter: _____________

7. Econometrics Continued

7. B. Econometrics continued on next page
7. Econometrics Continued

The following are the critical values for the Dickey-Fuller test statistic:

<table>
<thead>
<tr>
<th></th>
<th>1% Critical</th>
<th>5% Critical</th>
<th>10% Critical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3.682</td>
<td>-2.972</td>
<td>-2.618</td>
</tr>
</tbody>
</table>

(a) If real-estate appreciates at the rate of inflation, what does this imply about the stationarity of the nominal price series? Explain.

(b) If a housing bubble is characterized by housing prices appreciating “too fast” what might this imply about the level of integration of housing prices during the bubble?

(c) Using the graph of the levels of the price index, what can you postulate about the stationarity of the price series over time? Explain.

(d) Given the Dickey-Fuller test statistics presented in the three graphs above, what is the general level of integration of the Las Vegas house price index over the sample period? Explain.

(e) Consider the following time plot of the one-month differences in the Las Vegas housing index and the fitted values from the following regression:

\[ \Delta Price_t = \alpha + \sum_{k=1}^{10} \beta_k TIME^k + \epsilon \]

where \( TIME \) is a time index \( TIME \in [1, 268] \):

Discuss the predictability of the existence of a real estate “bubble” in Las Vegas given the tenth-order polynomial’s fitted values.
8. Answer the following question related to FINN6210 material. Show the details of your work! Answer all parts.

Consider the Black-Scholes model for an asset with no storage cost and no convenience yield.

a) What are the distributional assumptions for the future asset price? Be specific.

b) Briefly explain why the risk neutral valuation principle (RNVP) holds for the model.

c) Explain how you would use the RNVP to derive the B-S formula for the price of a call option on the asset. [I’m not asking you to actually derive the formula, though you should set up the computation and then carry it out as far as you are able].

d) Now consider a derivative that simply pays out \( S_T - K \) for some value \( K \) at some future time \( T \). Note that this payoff could be negative. Use the RNVP to derive the value of this position. [Hint: What does the B-S model imply about the risk-neutral expectation of the future asset price?]