University of North Carolina at Charlotte
Mathematical Finance Program
Comprehensive Exam

Spring, 2015

Directions: This exam consists of 6 questions. In order to pass the exam, you must answer each question. Where there have been multiple professors teaching the same class, we have provided a question from each (A. and B.). You should select the question most familiar for you, but don't answer both. <u>If you do answer both, only A. will be graded.</u>

Do not write your name on your answer sheet or on any exam page other than this cover sheet. You have been assigned an identification letter below. Write this identification letter on each of your answer pages *and* on each of the test sheets.

<u>Please note that we will not accept any answer sheet pages with your name on them; if your name is on the sheet we will not grade it.</u>

Your identification letter for this exam is: ________________________

Academic Honesty Statement: All students must comply with University policies on academic integrity. Any student violating these policies, as defined on pages 262-263 of the Graduate Catalog, will be referred to the University administration for disciplinary action. Sanctions for academic dishonesty include, but are not limited to, failure of this exam, suspension, or expulsion from the University. By signing below you certify that the answers you place on this exam are your own and that you have not received help from others.

I hereby certify that the work on this exam is my own. I also certify that I am aware that the test cannot be turned in after 4:00 pm and that I must place my exam identification letter on my exam sheets.

Student name (print) __________________________________

Student Signature: ____________________________________

Exam Identification Letter: ____________

Exam Policy for UNC Charlotte Master of Science in Mathematical Finance Program (Spring 2015)

1. Professors/proctors have the right to assign seats for students. If seats are not assigned, students need to spread out in the classroom.
2. During exam, students can use the materials allowed or provided by the professors. No other materials are allowed.
3. Only pen, pencil, eraser and a simple calculator are allowed during exam.
4. Students must use their own calculators. If a student wants to borrow a calculator, he or she can only borrow from the professor/proctor if available.
5. Students cannot use scratch paper or paper with notes/formulas during exam unless the professors/proctors allow them to do so.
6. Students will be given sufficient time to use the restroom before exam starts. ONLY in an urgent situation where the student must go to the restroom, he or she needs to obtain approval from the professor/proctor before leaving the exam room.

The above exam policy is consistent with the Code of Student Academic Integrity of the University of North Carolina at Charlotte. Please make sure to review carefully the entire policy at this link: http://legal.uncc.edu/policies/up-407.

The program has a zero tolerance policy toward any violations of the UNC Charlotte Code of Student Academic Integrity and the exam policies set forth by the professors. Professors/Proctors have the full right to ask the student who violates the academic integrity standards leave the exam room immediately. Any student, who violates the academic integrity standard and is consequently suspended or terminated, will not be readmitted to the program through reinstatement.

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1. **There were two professors who taught FINN/ECON6203. We have provided one question from each professor below. Please answer either question A <u>OR</u> question B below. Do not answer both A AND B.**

A. The 6-month, 12-month, 18-month, and 24-month zero rates are 2%, 3%,4% and 5% with continuous compounding in today's interest rate curve.

(a). Compute the implied forward rate for one-year period starting from1-year later.

(b). Construct an arbitrage opportunity if the forward rate over this time period (as in [a]) is quoted as 6% in the market.

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B. Consider a one-period market with 4 × 3 payoff matrix D [securities are in columns], and price vector p. Solving $D'\psi = p$, you find that

$$\psi = [1+x, 2-x, 2+2x, x]'$$

is a solution for all real numbers x. Answer the following questions about this market, carefully explaining your reasoning.

(a) Is this market complete?

(b) Does this market satisfy the Law of One Price?

(c) Does this market admit arbitrage?

(d) Does a risk-neutral measure exist? If so, calculate it.

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2. **There were two professors who taught FINN/ECON6219. We have provided one question from each professor below. Please answer either question A <u>OR</u> question B below. Do not answer both A AND B.**

A.

Let z_t be distributed $N(0,1)$ for all t, and assume that z_t is independent of z_{t+s} for all $s \neq 0$. Note that it follows that $\mathrm{E}(z_t^3) = 0$, $\mathrm{E}(z_t^4) = 3$, and $\mathrm{E}(z_t^5) = 0$ for all t.

Answer the following questions.

(a) Let $y_t = z_t$ and $x_t = z_t^2$. If you fit the simple linear regression model

$$y_t = \alpha + \beta x_t + e_t$$

by ordinary least squares (OLS), what should the OLS estimate of β converge to as the sample size goes to infinity? Your answer should be a number.

(b) Let $y_t = z_t$ and $x_t = z_t^3$. If you fit the simple linear regression model

$$y_t = \alpha + \beta x_t + e_t$$

by ordinary least squares (OLS), what should the OLS estimate of α converge to as the sample size goes to infinity? Your answer should be a number.

(c) Now suppose that

$$y_t = (1 + z_{t-1}^2) z_t$$

Find the expected value, variance, and first-order autocorrelation coefficient of y_t. Your answers should be numbers.

(d) Next suppose that

$$y_t = y_{t-1} + y_{t-1}^2 z_t$$

Find expressions for $\mathrm{E}(y_{t+1}|y_t)$ and $\mathrm{Var}(y_{t+1}|y_t)$.

(e) Finally suppose that

$$y_t = z_{t-1}^2 z_t^3$$

Is y_t a white-noise process? Explain why or why not. Be as specific as possible.

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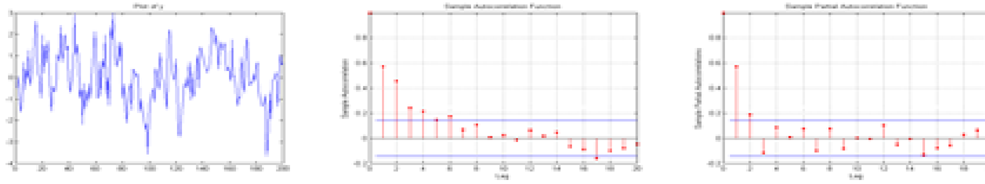
B.

1. Let $y_t = \varepsilon_t + \theta\varepsilon_{t-1}$ where θ is a real number and ε_t is a white noise series with zero mean and variance σ^2. Derive the autocovariance function for y_t. Is y_t covariance stationary?

2. Suppose you are modeling a time series y_t. Given the information below, determine the most appropriate model. Carefully explain your reasoning.



ARIMA(0,0,2) Model:

Conditional Probability Distribution: Gaussian

Parameter	Value	Standard Error	t Statistic
Constant	0.0786377	0.129656	0.606511
MA{1}	0.46012	0.069325	6.63714
MA{2}	0.389489	0.0729489	5.3392
Variance	0.983148	0.0935436	10.51

ARIMA(1,0,1) Model:

Conditional Probability Distribution: Gaussian

Parameter	Value	Standard Error	t Statistic
Constant	0.0248477	0.053603	0.46355
AR{1}	0.735222	0.0910338	8.07637
MA{1}	-0.239152	0.129295	-1.84966
Variance	0.967292	0.0934357	10.3525

ARIMA(1,0,2) Model:

Conditional Probability Distribution: Gaussian

Parameter	Value	Standard Error	t Statistic

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Constant	0.0432363	0.0848859	0.509347
AR{1}	0.500583	0.155049	3.22854
MA{1}	0.0058902	0.158982	0.0370495
MA{2}	0.216994	0.113349	1.9144
Variance	0.951195	0.0886284	10.7324

ARIMA(2,0,0) Model:

Conditional Probability Distribution: Gaussian

Parameter	Value	Standard Error	t Statistic
Constant	0.030646	0.0695802	0.440442
AR{1}	0.467078	0.0728941	6.40762
AR{2}	0.189089	0.0758313	2.49354
Variance	0.957899	0.0912966	10.4922

ARIMA(2,0,1) Model:

Conditional Probability Distribution: Gaussian

Parameter	Value	Standard Error	t Statistic
Constant	0.0608408	0.127856	0.475855
AR{1}	-0.301839	0.0784968	-3.84523
AR{2}	0.607259	0.0624218	9.7283
MA{1}	0.852194	0.0842701	10.1126
Variance	0.917188	0.0852319	10.7611

AR Terms	MA Terms	AIC	BIC
0.0	2.0	462.9797	481.7924
1.0	1.0	463.0377	481.8504
1.0	2.0	428.4640	451.9800
2.0	0.0	466.0009	484.8136
2.0	1.0	464.8937	488.4096

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3. Answer the following question related to FINN 6210 material. Show the details of your work! Answer all parts in this question.

Suppose the S&P500 index is currently 2000, its dividend yield is 2%, and the constant continuously compounded risk-free interest rate is 4%.

a. Find the current futures price for the October (6 months from now) futures contract on the index.

b. Suppose the current October futures price is actually trading for 2,050.00 on the open market. Carefully explain a way to exploit this for arbitrage. Remember that a futures contract is for delivery of $250 times the index. For simplicity, ignore all transactions costs.

c. Now consider a 4-month European call option on the October futures contract, with strike price 2000. Under what circumstances would the call be exercised, what would be the payoff, and when would the payoff be received?

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4. Answer the following question related to FINN 6211 materials. Show the details of your work! Answer all parts (a),(b) and (c) in this question.

a) For each of the following statements, provide in your answer: (1) if you AGREE or DISAGREE with the statement, and (2) one to two sentences explanation for why (if you agree) or why not (if you disagree).

a1) A coupon-bearing bond is simply a portfolio of zero-coupon bonds.

a2) "Forward rates are poor predictors of the actual future rates that are realized. Consequently, they are of little value to an investor." Explain why you agree or disagree with this statement.

a3) "For a given yield and maturity, the lower the coupon, the lower the convexity of a bond."

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b) On 4/15/2015 we are given the following market prices: (1) 99.155 on a zero-coupon bond maturing 10/15/2015; (2) 98.687 on a zero-coupon bond maturing 4/15/2016; (3) 97.485 on a zero-coupon bond maturing 10/15/2016; (4) 96.364 on a zero-coupon bond maturing 4/15/2017; (5) 95.324 on a zero-coupon bond maturing 10/15/2017. Assume the market price of a 6% fixed-rate note maturing on 10/15/2017 (semiannual coupon payments) is 107.312 and markets are perfect (zero transaction costs and bid-ask spreads, no credit/liquidity risk). Calculate the price of the zero-coupon bond portfolio replicating the fixed rate note. Set up an arbitrage strategy.

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c) The table below gives the prices, durations, and convexities of three bonds.

COUPON	MATURITY	PRICE	DURATION	Convexity
2.50%	5 years	102.275	4.723	25.052
2.75%	10 years	100.103	8.971	86.130
3%	30 years	94.934	19.475	495.423

c1) What is the duration and convexity of a portfolio that is long $10mm face amount of each of the 5- and 10-year bonds?
c2) What portfolio of the 5- and 30-year bonds has the same price and duration as the portfolio of part a)?
c3) Which of the two portfolios has the greater convexity and why?

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5. Answer the following question related to MATH 6203 materials. Show the details of your work! Answer all parts in this question.

MATH 6203 Comprehensive 2015 Spring - Mingxin Xu

Problem on Stochastic Calculus for Finance. Let W_t be a standard one-dimensional Brownian motion, and a, b, c be positive constants. X_t is a stochastic process with dynamics

$$dX_t = (a - bX_t)dt + cdW_t.$$

Question:

1. Find the dynamics of e^{r_t}.
2. Let $f : \mathbb{R} \to \mathbb{R}$ be a deterministic function. What partial differential equation does function f have to satisfy for $f(r_t)$ to be a martingale?

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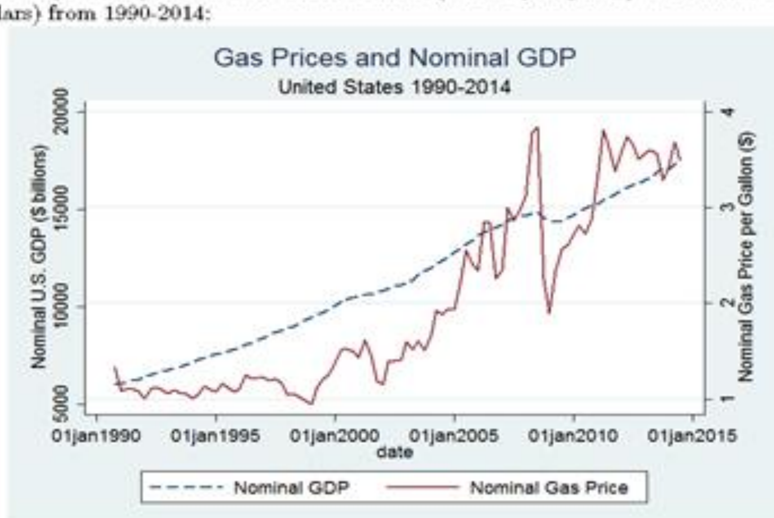
6. There were two professors who taught ECON 6213(A)/STAT6213(B). We have provided one question from each professor below. Some students might also select MATH 6201-Statistical Techniques in Finance (C). Please answer either question A <u>OR</u> question B <u>OR</u> question C below. Do not answer more than one.

A.

ECON6113

Consider nominal gasoline prices in the United States (dollars per gallon) and nominal U.S. GDP (in billions of dollars) from 1990-2014:



1. Define stationarity and why we are concerned with stationarity in time series models.
2. What can you say about the stationarity of these two variables using only the graph above?
3. Consider the following Dickey-Fuller test statistics on the levels and differences of the variables:

Variable	Levels	First Differences	Second Differences
Gas Prices	0.460	-83.81	
Nominal GDP	0.959	-16.57	-134.37

The critical values for the Dickey-Fuller tests are: -2.602 at the 1% level, -1.95 at the 5% level.

What do the tests indicate about stationarity of these two variables?

4. A colleague suggests that the high price of gasoline caused the growth of GDP during this period. Explain, with an equation, how you would test whether gasoline prices "cause" nominal GDP.
5. Interpret the results of the following causality tests obtained using SAS and the first differences of the data. Use $\alpha = 0.05$ as the significance level:

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Simple Summary Statistics						
Variable	Type	N	Mean	Standard Deviation	Min	Max
dgasprice	Dependent	95	0.02277	0.25385	-1.58800	0.66700
dnomgdp	Dependent	95	121.38842	84.14397	-293.10000	284.20000

Granger-Causality Wald Test			
Test	DF	Chi-Square	Pr > ChiSq
1	4	9.25	0.0551
2	4	8.75	0.0677

Test 1: Group 1 Variables:	dgasprice
Group 2 Variables:	dnomgdp

Test 2: Group 1 Variables:	dnomgdp
Group 2 Variables:	dgasprice

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B.

Question from STAT/ECON 6113

1. Data were collected for the purpose of studying the relationship between annual yields of several investment plans and the companies for the investments regarding two investment types: X_1 for Stocks and X_2 for Mutual Funds. For each investment type, the associated companies have been coded, so that -1 represents "from Company A", 0 represents "from Company B", 1 represents "from Company C" and 0.5 represents "a roughly equal mixture from Companies B and C", etc. Suppose the amount of investment is the same for each value of X_1 and X_2. The values of Y are coded as deviations in the yields compared to a traditional saving. The model $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$ is to be fit to the following data. Assume $\epsilon_i \overset{iid}{\sim} N(0, \sigma^2)$.

Observation	Y	X_1	X_2
1	-9	-1	-1
2	-3	-1	-1
3	20	1	1
4	16	1	1
5	0	-1	1
6	4	1	-1
7	5	0	0
8	3	0	0
9	8	0	1
10	12	1	0
11	-2	0	-1
12	-6	-1	0

Note, the total corrected sum of squares is 852 with 11 degrees of freedom, and

$$\mathbf{X}'\mathbf{X} = \begin{pmatrix} 12 & 0 & 0 \\ 0 & 8 & 2 \\ 0 & 2 & 8 \end{pmatrix}, (\mathbf{X}'\mathbf{X})^{-1} = \begin{pmatrix} 1/12 & 0 & 0 \\ 0 & 2/15 & -1/30 \\ 0 & -1/30 & 2/15 \end{pmatrix}, \mathbf{X}'\mathbf{Y} = \begin{pmatrix} 48 \\ 70 \\ 54 \end{pmatrix}.$$

(a) Compute the reduction in the residual sum of squares for adding X_2 to a regression model that already contains X_1 and an intercept. (Note: the SSE for the model with just X_1 and an intercept is 239.5).

(b) Compute a 95% prediction interval for a single new Y value that would be observed if the process is run with $X_1 = 1$ and $X_2 = 0.5$.

(c) Calculate an estimate of the mean square for pure error in this model and perform a lack-of-fit test.

(d) Assume that you could collect the response Y for one more X_1, X_2 combination. Describe how you would go about making the selection of the values for X_1 and X_2 that would minimize the expected uncertainty associated with the 95% prediction intervals for this response as well as the ones for the other 12 (X_1, X_2) locations. Note, you do not need to do this calculation, just describe how you would do it.

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C.

1. a) For the MA(1) process $Y_t - \mu = \epsilon_t - \theta\epsilon_{t-1},\ t = 1, 2, \ldots$ where ϵ_t is Gaussian WhiteNoise (0, σ^2) sequence,

i) Derive the mean, variance and ACF of Y_t.

ii) Derive the estimators of μ and θ.

b) Derive the ACF of the MA(2) process $Y_t = \mu + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2}$ where ϵ_t is Gaussian WhiteNoise (0, σ^2) sequence.

c) For the AR(1) process $X_t - \mu = \phi(X_{t-1} - \mu) + \epsilon_t,\ \ t = 1, 2, \ldots$ where the initial random variable X_0 is fixed and $0 < \phi < 1$ and ϵ_t be Gaussian WhiteNoise(0,1) sequence, represent X_t as an MA(∞) process.

d) Let ϵ_t be Gaussian WhiteNoise(0,1). For an ARCH(1) model, $Y_t = \epsilon_t\sqrt{\alpha_0 + \alpha_1 Y_{t-1}^2},\ \ t = 1, 2, \ldots, n$ where $\alpha_0 > 0$ and $0 \leq \alpha_1 < 1$, derive the conditional least squares estimators of (α_0, α_1).